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SOLUTION OF INVENTORY MODEL WITH SPACE CONSTRAINT AND FUZZY COST COMPONENTS BY FUZZY OPTIMIZATION TECHNIQUE USING NEAREST INTERVAL APPROXIMATION

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ABSTRACT

This paper discusses an Economic Order Quantity (EOQ) model with shortage and space constraint where the setup cost, the holding cost, the shortage cost are considered as fuzzy numbers. The fuzzy parameters are then transformed into corresponding interval numbers. Minimization of the interval objective function (obtained by using interval parameters) has been transformed into a classical multi-objective EOQ problem. The order relation that represents the decision maker's preference among the interval objective function has been defined by the right limit, left limit ,center and half –width of an interval. This concept is used to minimize the interval objective function. The problem has been solved by fuzzy programming technique. Finally, the proposed method is illustrated with a numerical example.

KEYWORDS: Inventory, Interval Number, Space Constraint, EOQ, Fuzzy Sets, Fuzzy Optimization Technique, Multi-Objective Programming.

INTRODUCTION

In traditional mathematical problems, the parameters are always treated as deterministic in nature. However, in practical problem, uncertainty always exists. In order to deal with such uncertain situations fuzzy model is used [1],[15]. in such cases, fuzzy set theory, introduced by Zadeh [14] is acceptable. There are several studies on fuzzy EOQ model. Lin et al. [7] have developed a fuzzy model for production inventory problem. Katagiri and Ishii[5] have proposed an inventory problem with shortage cost as fuzzy quantity.

This paper discusses a fuzzy EOQ model with shortage together with the space constraint. Demand, Holding cost, ordering cost, shortage cost are taken as triangular fuzzy numbers, and expression for fuzzy cost is established. For minimizing the cost function we transformed the fuzzy objective function into interval objective function. Now, this single objective function is then converted to multi-objective problem by defining left limit, right limit and center of the objective function. This multi- objective is then solved by fuzzy optimization technique. Linear membership function is considered here. This model is illustrated by a numerical example.

The article is organized as follows: In section 1 preliminary definitions of fuzzy set, interval number, basic interval arithmetic optimization in interval situation and nearest interval approximation is briefly described. Section 2 contains model formulation. The fuzzy optimization technique is section 3. In section 4 the process is illustrated by a numerical example and in the last section the entire work is concluded.

PRELIMINARIES

Definition 1:

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1]. (i.e) A={(x, $\mu_A(x)$) ; x϶ X.}, here μ_A:X→ [0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A is called the membership value of x϶ X in the fuzzy set A.

Definition 2:

Let ℜ be the set of all real numbers. An interval, may be expressed as

$$\overline{a}=[a_L,a_R]=\{x: a_L \le x \le a_R, a_L \in \Re, a_R \in \Re\} \tag{1}$$

where a_L and a_R are called the lower and upper limits of the interval $\overline{a}$, respectively.

If $a_L = a_R$ then $\overline{a}$ = [a_L , a_R] is reduced to a real number a, where a= a_L= a_R. alternatively an interval $\overline{a}$ can be expressed in mean-width or center-radius form as $\overline{a}=\langle m(\overline{a}), w(\overline{a})\rangle$,where $m(\overline{a})=\frac{1}{2}(a_L+a_R)$ and $w(\overline{a})=\frac{1}{2}(a_R-a_L)$ are respectively the mid-point and half-width of the interval $\overline{a}$ .the set of all interval numbers in ℜ is denoted by I(ℜ).

Optimization in interval environment

Now we define a general non-linear objective function with coefficients of the decision variables as interval numbers as

$$\text{Minimize } \overline{Z}(x) = \frac{\sum_{i=1}^{n} [a_{L_i}, a_{R_i}] \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} [b_{L_i}, b_{R_i}] \prod_{j=1}^{n} x_j^{q_j}} \tag{2}$$

subject to x_j >0, j=1,2,…,n and x ϵ S⊂ℜ where S is a feasible region of x, $0 < a_{L_i} < a_{R_i}$, $0 < b_{L_i} < b_{R_i}$ and r_i ,q_j are positive numbers. Now we exhibit the formulation of the original problem (2) as a multi-objective non-linear problem.

Now $\overline{Z}(x)$ can be written in the form $\overline{Z}(x)=[Z_L(x), Z_R(x)]$

$$\text{where } Z_L(x) = \frac{\sum_{i=1}^{n} a_{L_i} \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{R_i} \prod_{j=1}^{n} x_j^{q_j}} \tag{3}$$

$$Z_R(x) = \frac{\sum_{i=1}^{n} a_{R_i} \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{L_i} \prod_{j=1}^{n} x_j^{q_j}} \tag{4}$$

The center of the objective function

$$Z_C(x) = \frac{1}{2}\left[Z_L(x) + Z_R(x)\right] \tag{5}$$

Thus the problem (2) is transformed in to

$$\text{minimize}\{Z_C(x), Z_R(x); x \in S\} \tag{6}$$

subject to the non-negativity constraints of the problem, where Z_C,Z_R are defined by (4) and (5).

Nearest Interval Approximation

According to Gregorzewski [3] we determine the interval approximation of a fuzzy number as: Let $\tilde{A}$ =(a_1,a_2,a_3) be an arbitrary triangular fuzzy number with a α-cuts [$A_L(\alpha)$,$A_R(\alpha)$] and with the following membership function

$$\mu_{\tilde{A}}(x) = \begin{Bmatrix} \frac{x-a_1}{a_2-a_1}; a_1 \le x < a_2 \\ \frac{a_3-x}{a_3-a_2}; a_2 < x < a_3 \\ 0; otherwise \end{Bmatrix} \tag{7}$$

Then by nearest interval approximation method, the lower limit C_L and upper limit C_R of the interval are

$$C_L = \int_0^1 A_L(\alpha)d\alpha = \int_0^1 \left[a_1 + (a_2 - a_1)\alpha\right]d\alpha$$

$$= \frac{a_1+a_2}{2} \tag{8}$$

$$C_R = \int_0^1 A_R(\alpha)d\alpha = \int_0^1 \left[a_3 - (a_3 - a_2)\alpha\right]d\alpha$$

$$= \frac{a_2+a_3}{2}$$

Therefore, the interval number considering $\tilde{A}$ as triangular fuzzy number is $\left[\frac{a_1+a_2}{2}, \frac{a_2+a_3}{2}\right]$.

Model Formulation

In this model, an inventory with shortage together with a space constraint is taken into account. The purpose of this EOQ model is to find out the optimum order quantity of inventory item subject to constraint by minimizing the total average cost. We discuss the model using the following notations and assumptions throughout the paper.

Notations:

C_1: Holding cost per unit time per unit quantity.

C_2: Shortage cost per unit time per unit quantity.

C_3: Setup cost per period

D: The total number of units produced per time period.

A-The space required by each unit(in sq.mt)

B-Maximum available ware house space (in sq.mt)

Q_1: The amount which goes into inventory

Q_2: The unfilled demand

Q: The lot size in each production run.

Assumption:

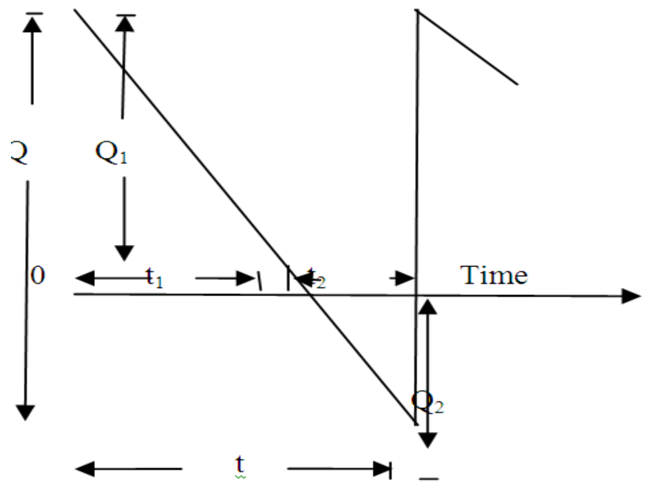
(i) Demand is known and uniform.

(ii) Production or supply of commodity is instantaneous.

(iii) Shortages are allowed.

(iv) Lead time is zero.

Inventory



The order level Q>0 which minimizes the average total cost (Q) per unit time subject to the space constraint is given by

$$\min C(Q) = \frac{1}{2}C_1\left(\frac{Q_1^2}{Q}\right) + \frac{1}{2}C_2\left(\frac{Q_2^2}{Q}\right) + C_3\left(\frac{D}{Q}\right)$$

Subject to : $AQ_1 \leq B$ (9)

Up to this stage, we are assuming that the demand, ordering cost, holding cost etc. as real numbers i.e .of fixed value. But in real life business situations all these components are not always fixed, rather these are different in different situations. To overcome these ambiguities we approach with fuzzy variables, where demand and other cost components are considered as triangular fuzzy numbers.

Let us assume the fuzzy demand $\tilde{D} = (D-\alpha, D, D+\beta)$ fuzzy holding cost $\tilde{C}_{1`} = (C_1-\alpha, C_1, C_1+\beta)$,fuzzy shortage cost, fuzzy ordering cost $\tilde{C}_3 = (C_3-\alpha, C_3, C_3+\beta)$ Replacing the real valued variables D,C_1,C_2 & C_3 by the triangular fuzzy variables $\tilde{D}, \tilde{C}_1, \tilde{C}_2, \tilde{C}_3$ we get,

$$\tilde{C}(Q) = \frac{1}{2}\tilde{C}_1\left(\frac{Q_1^2}{Q}\right) + \frac{1}{2}\tilde{C}_2\left(\frac{Q_2^2}{Q}\right) + \tilde{C}_3\left(\frac{\tilde{D}}{Q}\right)$$

Subject to: $AQ_1 \leq B$ (10)

Now we represent the fuzzy EOQ model to a deterministic form so that it can be easily tackled. Following Grzegorzewski [3] , the fuzzy numbers are transformed into interval numbers as

$\tilde{D} = (D-\alpha, D, D+\beta)$ =[D_L,D_R]

$\tilde{C}_{1`} = (C_1-\alpha, C_1, C_1+\beta)$ =[C_{1L},C_{1R}]

$\tilde{C}_{2`} = (C_2-\alpha, C_2, C_2+\beta)$ = [C_{2L},C_{2R}]

$\tilde{C}_3 = (C_3-\alpha, C_3, C_3+\beta)$ =[C_{3L},C_{3R}].

Using the above expression (10) becomes

$$\tilde{C}(Q) = \left[f_{L,} f_R \right] \quad (11)$$

Where,

$$f_L = \frac{1}{2}C_{1L}\left(\frac{Q_1^2}{Q}\right) + \frac{1}{2}C_{2L}\left(\frac{Q_2^2}{Q}\right) + C_{3L}\left(\frac{D_L}{Q}\right)$$

Subject to : $AQ_1 \leq B$ (12)

$$f_R = \frac{1}{2}C_{1R}\left(\frac{Q_1^2}{Q}\right) + \frac{1}{2}C_{2R}\left(\frac{Q_2^2}{Q}\right) + C_{3R}\left(\frac{D_R}{Q}\right)$$

Subject to : $AQ_1 \leq B$ (13)

The composition rules of intervals are used in these equations.

Hence the proposed model can be stated as

Minimize $\{ f_L(Q), f_R(Q)\}$, (14)

Generally, the multi-objective optimization problem(14), in case of minimization problem, can be formulated in a conservative sense from (3) as

Minimize $\{ f_C(Q), f_R(Q)\}$,

Subject to : $AQ_1 \leq B$

$Q \geq 0$. (15)

Where $f_C = \frac{f_L + f_R}{2}$.

Here the interval valued problem (14) is represented as

Minimize $\{ f_L(Q), f_C(Q), f_R(Q)\}$,

Subject to : $AQ_1 \leq B$

$Q \geq 0$ (16)

The expression (16) gives a better approximation than those obtained from (14).

Fuzzy Programming Technique for Solution

To solve multi-objective minimization problem given by (16), we have used the following fuzzy programming technique.

For each of the objective functions $f_L(Q)$, $f_C(Q)$, $f_R(Q)$,subject to the space constraint we first find the lower bounds L_L, L_C, L_R (best values) and the upper bounds U_L, U_C, U_R (worst values), where L_L, L_C, L_R are the aspired level achievement and U_L, U_C, U_R are the highest acceptable level achievement for the objectives $f_L(Q)$, $f_C(Q)$, $f_R(Q)$ respectively and $d_k = U_k - L_k$ is the degradation allowance for objective $f_k(Q)$, k=L,C,R. Once the aspiration levels and degradation allowance for each of the objective function has been specified, we formed a fuzzy model and then transform the fuzzy model into a crisp model. The steps of fuzzy programming technique is given below.

Step 1: Solve the multi-objective cost function subject to the constraint as a single objective cost function subject to the constraint using one objective at a time and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From step 2, we find for each objective, the best L_k and worst U_k value corresponding to the set of solutions. The initial fuzzy model of (10) can then be stated as, in terms of the aspiration levels for each objective, as follows: find Q satisfying $f_k \tilde{<} L_k, k = L, C, R$ subject to the space constraint non negativity conditions.

Step 4: Define fuzzy linear membership function $\left(\mu_{f_k}; k = L, C, R\right)$ for each objective function is defined by

$$\mu_{f_k} = \begin{cases} 1; f_k \le L_k \\ 1 - \dfrac{f_k - L_k}{d_k}; L_k \le f_k \le U_k \\ 0; f_k \ge U_k \end{cases} \quad (17)$$

Step 5: After determining the linear membership function defined in(17) for each objective functions following the problem (16) can be formulated an equivalent crisp model

Max α,

$\alpha \le \mu_{f_k}(x); k = L, C, R.$

$AQ_1 \le B$, $\alpha \ge 0$, $Q \ge 0$.

Numerical Example

In this section, the above mentioned algorithm is illustrated by a numerical example.

Here the parameters demand, ordering cost, holding cost and shortage cost are considered as triangular fuzzy numbers (TFN). After that, the fuzzy numbers are transformed into interval numbers using nearest interval approximation following [3].

Let C_1=5, C_2=25, C_3=100, D=5000,A=0.5 sq.mt, B=150sq.mt units.

Taking these as triangular fuzzy numbers we have,

$\tilde{C}_1$=(3,5,7), $\tilde{C}_2$=(21,25,31), $\tilde{C}_3$=(85,103,109) $\tilde{D}$=(4000,5000,6000).

The fuzzy numbers $\tilde{D}, \tilde{C}_1, \tilde{C}_2, \tilde{C}_3$ are transformed into interval numbers as,

=$[D_L, D_R]$=[4500,5500]

$\tilde{C}_1$=$[C_{1L}, C_{1R}]$=[4,6]

$\tilde{C}_2$= $[C_{2L}, C_{2R}]$=[23,28]

$\tilde{C}_3$=$[C_{3L}, C_{3R}]$=[94,106]

Individual minimum and maximum of objective functions f_L, f_C, f_R are given below:

Table 1

Objective Functions	Optimize f_L	Optimize f_C	Optimize f_R
f_L	$f_L^{'}$=1780.32	$f_L^{''}$=1781.34	$f_L^{'''}$=1783.96

f_C	$f_C{}'$ =2123.43	$f_C{}''$ =2122.28	$f_C{}'''$ =2123.19
f_R	$f_R{}'$ =2482.43	$f_R{}''$ =2478.88	$f_R{}'''$ =2477.87

Now we calculate

$$L_L = \min\left(f_L{}', f_L{}'', f_L{}''' \right) = 1780.32 \quad U_L = \max\left(f_L{}', f_L{}'', f_L{}''' \right) = 1783.96$$

$$L_C = \min\left(f_C{}', f_C{}'', f_C{}''' \right) = 2122.28 \quad U_C = \max\left(f_C{}', f_C{}'', f_C{}''' \right) = 2123.43$$

$$L_R = \min\left(f_R{}', f_R{}'', f_R{}''' \right) = 2477.87 \quad U_R = \max\left(f_R{}', f_R{}'', f_R{}''' \right) = 2482.43$$

Using the equation (18), we formulate the following problem as:

Max α

$$\frac{1}{2}(4)\left(\frac{Q_1{}^2}{Q}\right) + \frac{1}{2}(23)\left(\frac{Q_2{}^2}{Q}\right) + 94\left(\frac{4500}{Q}\right) + (3.64)\alpha \le 1783.96$$

$$\frac{1}{2}(5)\left(\frac{Q_1{}^2}{Q}\right) + \frac{1}{2}(25.5)\left(\frac{Q_2{}^2}{Q}\right) + 100\left(\frac{5000}{Q}\right) + (1.145)\alpha \le 2123.42$$

$$\frac{1}{2}(6)\left(\frac{Q_1{}^2}{Q}\right) + \frac{1}{2}(28)\left(\frac{Q_2{}^2}{Q}\right) + 106\left(\frac{5500}{Q}\right) + (4.563)\alpha \le 2482.43$$

(0.5)A ≤ 150;

α ≥0; Q ≥ 0. (18)

RESULTS AND DISCUSSIONS

The solutions obtained from (18) is given in table 2 and 3.

Table 2: Optimum Value of α

Maximum α
0.75003

Table 3: Optimum Results

$f_L{}^*$	$f_C{}^*$	$f_R{}^*$	Q^*	Q_1^*
1781.2	2122.3	2479.0	382.9	300.0

Table 4: Comparison Table

Model	C_1	C_2	C_3	Q*	Q_1*	C*(Q)	α
Crisp	5	25	100	384.7	300	2117.7	--
Crisp	5	20	50	353.6	282.8	1414.2	--
Crisp	3	20	50	358.5	300	1169.4	--
fuzzy	[4,6]	[23,28]	[94,106]	382.9	300	[1781.2,2479.0]	0.7500

CONCLUSIONS

In this paper, we have presented an inventory model with shortage together with the space constraint, where carrying cost, shortage cost, ordering or setup cost and demand are assumed as fuzzy numbers instead of crisp or probabilistic in nature to make the inventory model more realistic. At first, expression for the total cost is developed containing fuzzy parameters. Then each fuzzy quantity is approximated by interval number. After that the problem of minimizing the cost function subject to the constraint is transformed into a multi-objective inventory problem subject to the constraint, where the objective functions are left limit, right limit and the center of the interval function. Fuzzy optimization technique is then used to found out the optimal results. A numerical example illustrates the proposed method.

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